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Field synergy optimization and enhanced heat transfer by multi-longitudinal vortexes flow in tube

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Abstract

The field synergy equation for steady laminar convection heat transfer was derived by conditional variation calculus based on the least dissipation of heat transport potential capacity. The optimum velocity field with the best heat transfer performance and least flow resistance increase can be obtained by solving the synergy equation. The numerical simulation of laminar convection heat transfer in a straight circular tube shows that the multi-longitudinal vortex flow in the tube is the flow pattern that enhances the heat transfer enormously. Based on this result, a novel enhanced heat transfer tube, the discrete double-inclined ribs tube (DDIR-tube), is developed. The flow field of the DDIR-tube is similar to the optimal velocity field. The experimental results show that the DDIR-tube has better comprehensive heat transfer performance than the current heat transfer enhancement tubes. The present work indicates that new heat transfer enhancement techniques could be developed according to the optimum velocity field.

Keywords: Enhanced heat transfer; Optimization; Calculus of variations; Field synergy; Multi-longitudinal vortexes; Discrete double-inclined ribs tube

1. Introduction

Heat transfer enhancement techniques have been developing rapidly and there have been broad applications in the past several decades. Most of the techniques, however, are based on experiences and experiments. For instance, an enhancement element could be designed according to researchers' experiences, and then its heat transfer correlations are obtained by experiments or numerical calculations [1–4]. The performances of convective heat transfer are dependent on the velocity and temperature fields, so that modifying the velocity field is the most direct approach to enhance convective heat transfer. It is very difficult to find the best velocity field that improves the heat transfer enhancement the most even for an experienced researcher. The current research method on heat transfer enhancement is technical and lacks an optimization theory to guide the design for various enhancement techniques.

Guo and his colleagues [5,6] investigated the 2D boundary layer flow over a flat plate from the point of view of the relation between flow field and temperature field. The energy equation was regarded as a conduction equation where the convection term is taken as a heat source. The wall heat flux is equal to the overall strength of the heat sources inside the thermal

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Nomenclature

A	anti-temperature, K
C_0, C_1	Lagrange multipliers
C_Φ	coefficient of the synergy force, N/m^{2}K
c_p	specific heat, J/kg K
D	hydraulic diameter
F	volume force, N/m^3
f	frictional factor
J	Lagrange function
J_Φ	total viscous dissipation, W
J_Ψ	dissipation of heat transfer potential capacity, W/K
k	thermal conductivity, W/m K
L	total length of tube, m
Nu	Nusselt number
P	pressure, N/m^2
Pr	Prandtl number
Re	Reynolds number
S	surface area, m^2
T	temperature, K
U	velocity vector, m/s
V	volume, m^3
u, v, w	velocity components in x, y, z direction, m/s
x, y, z	coordinates, m
y	ratio of twisted

Greek symbols

Γ	Surface on domain
Φ	viscous dissipation function, W/m^3
Ω	domain
μ	dynamic viscosity, Pa s
ρ	density, kg/m^3

Subscripts

c	cool
e	enhancement tube
f	fluid
h	high
int	inlet
out	outlet
s	smooth tube
m	mean
w	wall
x, y	x, y direction

boundary layer. This implies that the convection heat transfer can be enhanced by increasing the quantity of the integral of the convection terms (heat sources) over the thermal boundary layer. Based on their analysis, they proposed the principle of field synergy for convective heat transfer enhancement. It states that the better the coordination of velocity and heat flow fields is, the higher is the heat transfer rate of convection at given velocity and temperature gradient. The complete synergy between velocity and temperature fields occurs when the temperature gradient is always parallels to the velocity vector. It should be noted that complete synergy between velocity and temperature fields can never be realized, the temperature gradient is almost normal to the velocity vector for most convective heat transfer problems, such as convections in tubes. Therefore, the convection heat transfer can be greatly enhanced by improving the coordination between the fluid and heat flow fields.

The flow field, however, influences the temperature field according to the energy equation. We can try to make the source term as large as possible. This paper will derive the optimum velocity field equation for steady laminar convection heat transfer. The optimum velocity field in tube will be found out by numerically solving the optimum velocity field equation. A novel enhanced heat transfer tube is developed according to the optimum velocity field.

2. Optimization of flow field for laminar convection heat transfer

Mechanical work should be paid to maintain the fluid flow because of the viscous dissipation. The viscous dissipation function of Newtonian fluid is,

$$\Phi = \mu\left[2\left(\frac{\partial u}{\partial x}\right)^2 + 2\left(\frac{\partial v}{\partial y}\right)^2 + 2\left(\frac{\partial w}{\partial z}\right)^2 + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)^2\right] \quad (1)$$

where u, v, and w are the velocity components along x, y, z directions, respectively. The mechanical work maintaining the fluid flow equals to the integral of the viscous dissipation function over the whole domain, that is,

$$J_\Phi = \int_\Omega \Phi \, \mathrm{d}V \quad (2)$$

Heat transport is a typical irreversible process, and something should be also paid out when heat is transferred from high temperature to low temperature. It is just like that mechanical work must be paid to sustain the flows from inlet to outlet.

Boit defined the thermal potential and thermal dissipation function and presented a variation calculus to heat transfer that leads to a Lagrange Equation in the generalized coordinates [7,8]. However, the heat transport equation by Boit is only a quasi-variation formulation and is given for an approximate solution. The physical meaning of the heat transport equation is somewhat vacuous [9]. The heat dissipation function defined by Boit seems not reasonable for heat transport process. In other words, it could not be said that heat is dissipated in a heat transfer process.

Guo et al. [10] defined the heat transfer potential capacity and the dissipation function. Their physical meanings are the overall heat transfer capability and the dissipation rate of the heat transfer capacity, respectively. The dissipation function of the heat transfer potential capacity is the payout for heat transport. The total dissipation of the heat transfer potential capacity J_Ψ in heat transfer domain Ω is

$$J_\Psi = \frac{1}{2}\int_\Omega k(\nabla T)^2\,\mathrm{d}V \tag{3}$$

It is proved to be valid for steady heat conduction optimization problem. According to the principle, the most effective transport path can be constructed by inserting the given amount of high conductivity material in the conduction domain [10].

For laminar convection in tube with constant thermophysical properties and without internal heat source, the dissipation of heat transfer potential capacity corresponds to the heat transfer performances. Unlike the steady heat conduction, convection heat transfer can not be optimized only by the minimal dissipation of heat transfer potential capacity because the velocity field influences the heat transfer enormously. Xia [11] took the dissipation of heat transfer potential capacity as the evaluation criterion for the heat transfer performance, and used viscous dissipation to evaluate the loss of mechanical work in the flow process. He indicated that the optimum velocity field could be obtained by the variation calculus. Therefore, for the laminar convection heat transfer with constant thermophysical properties and without internal heat generation, the optimum velocity field could be obtained by minimizing the dissipation of heat transport potential capacity under the condition of certain viscous dissipation. This is a functional problem which can be solved by the conditional variation calculus. Meng [12] studied this functional problem by variational principles.

The condition for the least dissipation of heat transfer potential capacity is

$$\delta J_\Psi = 0 \tag{4}$$

The constant viscous dissipation or mechanical work payout can be expressed as

$$\delta J_\Phi = 0 \tag{5}$$

For the flow boundary condition, the inlet velocity is given or the flow is assumed to be fully developed, which is described by variation symbol as

$$\delta U|_{\mathrm{in}} = 0 \quad \text{or} \quad \delta U|_{\mathrm{in}} = \delta U|_{\mathrm{out}} \tag{6}$$

For the isothermal or constant heat flux thermal boundary condition, there is

$$\delta T|_{\mathrm{w}} = 0 \quad \text{or} \quad \delta(\lambda\nabla T)|_{\mathrm{w}} = 0 \tag{7}$$

In addition, the convective heat transfer must satisfy the continuity equation,

$$\nabla\cdot(\rho\mathbf{U}) = 0 \tag{8}$$

and the energy equation,

$$\lambda\nabla^2 T - \rho c_p \mathbf{U}\cdot\nabla T = 0 \tag{9}$$

To get the optimal flow field, we need to establish a Lagrange function which includes the objective and constraint functions, then do the variation calculus to the Lagrange function and solve the equations of Lagrange multipliers and the constraint functions [13]. The established Lagrange function is,

$$J^* = \int_\Omega \left\{\frac{1}{2}\lambda(\nabla T)^2 + C_0\Phi + A(\lambda\nabla^2 T - \rho c_p\mathbf{U}\cdot\nabla T) + C_1\nabla\cdot\mathbf{U}\right\}\mathrm{d}\Omega \tag{10}$$

where C_0, A, C_1 are Lagrange multipliers, C_0 is required to be constant, A and C_1 are functions of $\mathbf{U}$, T, and the position. We could derive the following Eq. (11)–(14) by the variations of J^* with respect to T and $\mathbf{U}$,

$$\lambda\nabla^2 A + \rho c_p\mathbf{U}\cdot\nabla A - \lambda\nabla^2 T = 0 \tag{11}$$

$$\int_\Gamma \{[\lambda\nabla T - (\lambda\nabla A + \rho c_p\mathbf{U}A)]\delta T + A\delta\nabla T\}\,\mathrm{d}\vec{S} = 0 \tag{12}$$

$$-2C_0\mu\nabla^2\mathbf{U} - \rho c_p\mathbf{U}\nabla A - \nabla C_1 = 0 \tag{13}$$

$$\int_\Gamma (2C_0P + C_1)\delta\mathbf{U}\cdot\mathrm{d}\vec{S} = 0 \tag{14}$$

Eqs. (11) and (13) are the equations of Lagrange multipliers. Eqs. (12) and (14) are the boundary conditions of Eqs. (11) and (13) respectively.

Eqs. (11) and (12) show that A is of temperature dimension, and it is defined as anti-temperature because Eq. (11) is similar to an energy equation with a contrary velocity field, $-U$, and a heat generation item of $-\lambda\nabla^2 T$. On the wall surface, Eq. (12) can be rewritten as $A_{\mathrm{w}} = 0$ for isothermal wall, $(-\lambda\nabla A)_{\mathrm{w}} = (-\lambda\nabla T)_{\mathrm{w}}$ for constant heat flux, and $(-\lambda\nabla A)_{\mathrm{w}} = 0$ for adiabatic wall. At the inlets and outlets, Eq. (12) can be rewritten as $A_{\mathrm{in}} = 0$ for isothermal inlet and $(A\nabla T)_{\mathrm{in}} \approx (A\nabla T)_{\mathrm{out}}$ for fully-developed velocity and temperature.

According to the reference [14], C_1 in Eqs. (13) and (14) can be set as follows,

$$C_1 = -2C_0P \tag{15}$$

where P is pressure. Then Eq. (13) can be rewritten as,

$$\mu\nabla^2\mathbf{U} - \rho\mathbf{U}\cdot\nabla\mathbf{U} - \nabla P + \left(\frac{\rho c_p}{2C_0}A\nabla T + \rho\mathbf{U}\cdot\nabla\mathbf{U}\right) = 0 \tag{16}$$

Eq. (16) is similar to the momentum equation (Navier–Stokes equation), with an additional force,

$$\mathbf{F} = (\rho c_p/2C_0)A\nabla T + \rho\mathbf{U}\cdot\nabla\mathbf{U}$$
$$= C_\Phi A\nabla T + \rho\mathbf{U}\cdot\nabla\mathbf{U} \tag{17}$$

where constant C_Φ is related to the input viscous dissipation; and A satisfies Eq. (11) and boundary condition (12). Therefore, the governing equation for the optimal velocity field could be further reduced to

$$\mu\nabla^2\mathbf{U} - \rho\mathbf{U}\cdot\nabla\mathbf{U} - \nabla P + (C_\Phi A\nabla T + \rho\mathbf{U}\cdot\nabla\mathbf{U}) = 0 \tag{18}$$

The velocity field satisfying Eq. (18) will lead to the best performance of heat transfer. On the other words, any changes of the velocity field could only weaken the heat transfer when the viscous dissipation maintains constant. According to the principle of field synergy for convective heat transfer, the optimum velocity field is in better synergy with the temperature field. The Eq. (18) is defined as the field synergy equation for convective heat transfer and the optimum additional force $\mathbf{F} = C_\Phi A\nabla T + \rho\mathbf{U}\cdot\nabla\mathbf{U}$ is named as synergy force. Obviously, the synergy force is related to the velocity field and the temperature gradient field. It is a special force that drives the fluid flow in synergy with the heat transfer, that is, promotes the fluid to flow along the direction of the temperature gradient. The field synergy equation and synergy force are completely consistent with the suggestion by Guo [5] who indicated the concept of field synergy for convective heat transfer, i.e. the heat transfer could be enhanced most if the velocity and heat flux vectors are in the same direction. The space distribution of the synergy force is dependent on the characteristics of heat transport, i.e. the distribution of temperature T, anti-temperature A and velocity U. The potential effect of synergy force is to drive fluid to flow along the heat flux or in the reverse direction. However, because of the constraint of the actual boundary, the optimum flowing pattern governed by the field synergy equation is generally more complicated.

The field synergy equation for convective heat transfer is similar to but not an ordinary momentum equation (Navier–Stokes equation) because it contains a virtual additional force, the synergy force. The synergy force of the field synergy equation was derived by conditional variation calculus. It does not exist in nature. Therefore, the optimum velocity field indicates the basic characteristics of the flow pattern that benefits the heat transfer mostly. If an enhancement element could create a velocity field similar to the optimum velocity field, it will have a very satisfactory performance in heat transfer and flow resistance. The most direct application of the synergy equation is to offer a guidance for selecting and designing an appropriate enhancement technique.

3. The optimum velocity field for laminar convection in circular tube

The laminar convection heat transfer enhancement in tubes is a classical problem with wide engineering applications. Circular tubes are commonly used as heat transfer elements in tube-shell heat exchangers. The optimum velocity field for laminar convection heat transfer in the tube can be obtained through solving the field synergy equation under a certain flow and thermal boundary conditions.

A straight circular tube with 20 mm in diameter and 30 mm in length is selected for numerical simulation. Only one half of the cross-section of the tube is analyzed due to symmetry. Both the flow and temperature are assumed to be fully developed and the wall is isothermal. The wall temperature is 310 K; the average temperature of the inlet fluid is 300 K; the inlet Reynolds number is 400.

FLUENT 6.0 is used to solve the field synergy equation numerically. The UDF function in FLUENT 6.0 is used for solving the anti-temperature Eq. (12). In the numerical simulation, the anti-temperature equation is solved synchronously with the continuity equation, velocity field synergy equation and energy equation. The algorithm of SIMPLEC is used to uncouple the pressure and velocity. The QUICK format is used for the divergence of both the field synergy and energy equations.

The numerical solutions with different constant C_Φ (corresponding to certain viscous dissipation work) result in different flow patterns. For a given C_Φ, a flow field can be numerically obtained, then, the viscous dissipation can be calculated. That is, C_Φ is related to viscous dissipation. The range of C_Φ is from -0.0005 to -0.02 for solving the field synergy equation in the straight circular tube. When $|C_\Phi|$ is small, four longitudinal vortexes occur in the cross-section, and the strength of the longitudinal vortexes increases with the increasing $|C_\Phi|$. When $|C_\Phi|$ is larger than a critical value, the number of the longitudinal vortexes in the cross-section change from four to eight. Furthermore, when $|C_\Phi|$ increases continuously to another higher value, the numerical calculation of the velocity field can not converge, which may be attributed to the transition to turbulence for very large $|C_\Phi|$. Fig. 1 shows a typical numerical result of the cross-sectional flow field ($Re = 400$, $C_\Phi = -0.01$). Compared with the fully-developed laminar convection heat transfer in a circular tube ($(fRe)_s = 64$, $Nu_s = 3.66$), the flow viscous dissipation is

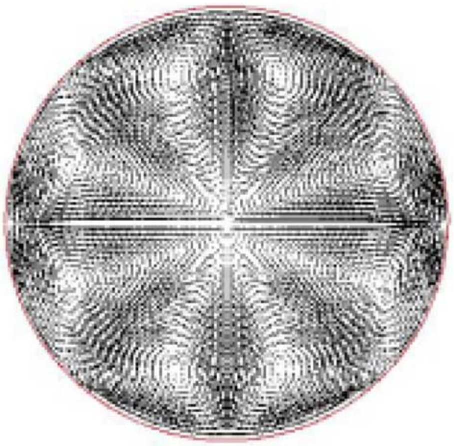

Fig. 1. Optimum flow field of laminar heat transfer in circular tube ($Re = 400$).

increased by 17%, the heat transfer rate (the Nusselt number) is increased by 313% in the case of Fig. 1.

The numerical analysis shows that multiple longitudinal vortex flow is the optimal flow pattern for laminar flow in tube. Therefore, construct a multi-longitudinal vortex flow can markedly enhance the convection heat transfer in tubes, which can guide the researcher to design the enhanced element.

4. A novel enhanced heat transfer tube

Based on the field synergy analysis for laminar heat transfer, a novel enhanced tube, discrete double-inclined ribs tube (DDIR-tube) [15], was developed. Fig. 2 is the photo of the DDIR-tube. Multi-longitudinal vortexes flow can be induced by the periodical surface variation (discrete double-inclined ribs). The outer diameter of the DDIR-tube for experiment is 20 mm. There are three pairs of double-inclined ribs in the cross-section, the pitch is 12 mm, the rib length in axial direction is 6 mm, the inclined angle is about 45°, and the inner rib height is 0.85 mm.

The numerical simulation by the field synergy equation shows that the optimal velocity field of the laminar heat transfer in tube is 2–4 multi-longitudinal vortex pairs. The further numerical analysis also shows that the more the multi-longitudinal vortexes in tube, the more remarkable the turbulent heat transfer enhancement is. Due to the limitation of manufacture, the number of the discrete double-inclined rib pairs in the same cross-section should be determined by the tube diameter. For the commonly used tubes of Φ 19–25 mm, three to seven discrete double-inclined ribs pairs are recommended when applying the pressing process.

Fig. 3 depicts the numerical cross-sectional flow field in the DDIR-tube for $Re = 1000$ using the RNG k–ε model. It shows that strong multi-longitudinal vortex flow is induced by the discrete double-inclined ribs on the internal wall. The cross-sectional flow pattern of the DDIR-tube is similar to the optimum flow field obtained by solving the field synergy equation.

The experiments are performed to clarify the heat transfer and flow characteristics of the DDIR tube. The tested tube length is 2 m. The fluid outside the DDIR-tube is deionized water, and the fluid inside the tube is 22# lubricating oil. The experimental results of heat transfer and flow resistance for the DDIR-tube are shown in Fig. 4. For Re ranging from 500 to 2300, the Nusselt numbers are increased by 250–650% with a resistance increase of 120–300% compared with those of laminar convection in a circular tube ($L/D = 300$) with inlet effect considered. For Re ranging from 2300 to 15000, the Nusselt numbers are increased by 240–110% with a resistance increase of 130–210% compared with those of transitional and turbulent heat transfer in a circular tube ($L/D = 300$).

The comparison of the comprehensive performances is shown in Fig. 5. The subscript 'e' means enhanced tube, and 's' means smooth circular tube. It is seen from Fig. 5 that the DDIR-tube is of good performance of heat transfer enhancement. For example, comparing

Fig. 2. Photo of the novel enhanced heat transfer tube.

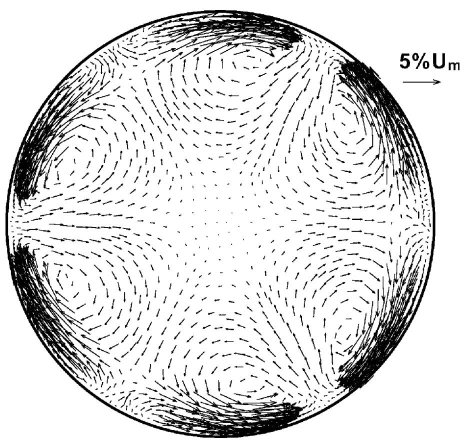

Fig. 3. Numerical cross-sectional flow fields in the DDIR-tube.

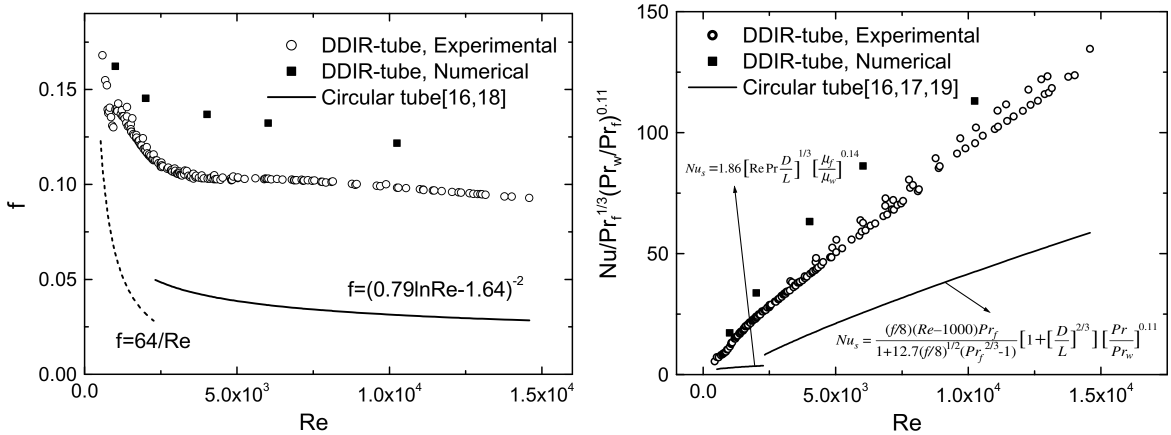

Fig. 4. Experimental and numerical results of heat transfer and flow resistance for DDIR-tube.

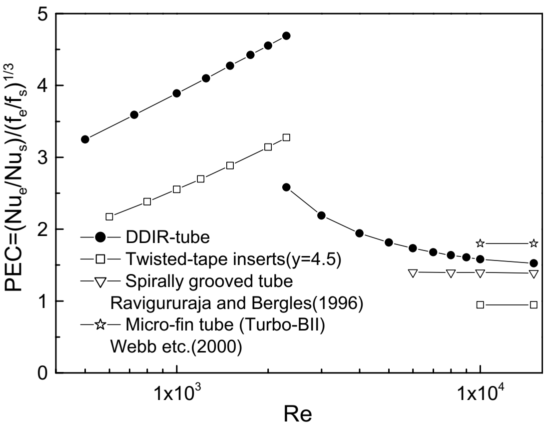

Fig. 5. PEC for different enhanced tubes.

with laminar heat transfer in a circular tube ($L/D = 300$) where the inlet effect is considered [16–19], the heat transfer enhancement criteria PEC, defined as $(Nu_e/Nu_s)/(f_e/f_s)^{1/3}$, is 2.0–3.3 for tubes with twisted tape inserts ($y = 4.5$) [20], however, the PEC of the DDIR-tube is 3.2–4.7. Circular tube with twisted tape inserts are commonly regarded as the best enhanced element in the low Reynolds number region. Compared with the circular tube with twisted tape inserts, PEC of the DDIR-tube is 40–60% increased, the Nusselt number is about 10% increased and the frictional resistance is 40–60% decreased in the Reynolds number region of 500–2300. The comprehensive performances of the DDIR-tube are also better than that of the spirally grooved tube [21], but are slightly worse than that of the micro-fin tube (Turbo-BII) [22]. The DDIR-tube is easier to be manufactured and could have good performance for counteracting scaling due to the induced multiple longitudinal vortex flow.

5. Conclusions

The field synergy equation of steady laminar convection heat transfer was derived with the least dissipation of heat transport potential capacity as the optimum target under the condition of given viscous dissipation by the variational principle for the first time.

Numerical solution of the field synergy equation of laminar convection heat transfer in a straight circular tube together with other governing equations indicates that the multi-longitudinal vortex flow is the best way for heat transfer enhancement in laminar convection in tubes. The optimum velocity field has excellent performances of heat transfer and resistance.

A novel tube, DDIR-tube, was developed. The multi-longitudinal vortex flow can be induced by the discrete double-inclined ribs on the internal wall tube, which is similar to the optimal flow pattern given by the synergy equation. The numerical calculation and experiment show that the comprehensive performance of enhanced laminar heat transfer in DDIR-tube are better than that of the currently-known enhancement techniques. The Nusselt number can be increased by 250–650% with a resistance increase of 120–300% compared with those of laminar heat transfer in a circular tube ($L/D = 300$) with inlet effect considered for $Re = 500$–2300. And the Nusselt number can be increased by 240–110% with a resistance increase of 130–210% compared with that of transitional and turbulent heat transfer in a circular tube ($L/D = 300$) for $Re = 2300$–15000.

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